

# Cosmology with Independently Varying Neutrino Temperature and Number

Richard Galvez<sup>1,2</sup> and Robert J. Scherrer<sup>1</sup>

<sup>1</sup>*Department of Physics and Astronomy, Vanderbilt University, Nashville, TN 37212, USA*

<sup>2</sup>*Department of Physics and Astronomy, Fisk University, Nashville, TN 37208, USA*

We consider Big Bang nucleosynthesis and the cosmic microwave background in a model in which both the neutrino temperature and neutrino number are allowed to vary from their standard values. The neutrino temperature is assumed to differ from its standard model value by a given factor from Big Bang nucleosynthesis up to the present. In this scenario, the effective number of relativistic degrees of freedom,  $N_{\text{eff}}^{\text{CMB}}$ , derived from observations of the cosmic microwave background is not equal to the true number of neutrinos,  $N_\nu$ . We determine the element abundances predicted by Big Bang nucleosynthesis as a function of the neutrino number and temperature, converting the latter to the equivalent value of  $N_{\text{eff}}^{\text{CMB}}$ . We find that a value of  $N_{\text{eff}}^{\text{CMB}} \approx 3$  can be made consistent with  $N_\nu = 4$  with a decrease in the neutrino temperature of  $\sim 5\%$ , while  $N_\nu = 5$  is excluded for any value of  $N_{\text{eff}}^{\text{CMB}}$ . No observationally-allowed values for  $N_{\text{eff}}^{\text{CMB}}$  and  $N_\nu$  can solve the lithium problem.

## I. INTRODUCTION

Cosmological observations currently provide some of the most robust probes of physics beyond the Standard Model of particle physics. Within the context of cosmology, the analysis of the cosmic microwave background (CMB) temperature anisotropies and the comparison between the observed light element abundances and those predicted by Big Bang nucleosynthesis (BBN) are two of the most robust methods to test theoretical predictions.

One of the first such cosmological constraints was the placement of an upper limit on the number of neutrinos inferred from the abundance of  $^4\text{He}$  using BBN predictions [1] to be  $N_\nu \leq 5$ . More recently, the most precise limits on the effective number of relativistic species, which we will denote  $N_{\text{eff}}^{\text{CMB}}$ , have come from CMB observations such as those from the WMAP [2] and Planck [3] experiments. This parameter,  $N_{\text{eff}}$ , is an estimate of the total energy density contained in relativistic particles at recombination, parametrized in terms of the number of effective two-component neutrinos. Recent results from the PLANCK collaboration combined with other astrophysical data give [3]

$$N_{\text{eff}}^{\text{CMB}} = 3.15 \pm 0.23, \quad (1)$$

where this value and its best-fit estimate are inferred in a Bayesian fashion by combining the Planck measurements with other astrophysical and cosmological data sources. While  $N_{\text{eff}}^{\text{CMB}}$  includes any particles that are relativistic at recombination, we focus on the case that such “dark radiation” consists entirely of neutrinos and leave more exotic possibilities to future work.

It is often assumed that CMB measurements of  $N_{\text{eff}}^{\text{CMB}}$  probe the number of neutrinos  $N_\nu$  directly, but this need not be the case. CMB observations are generally only sensitive to the total neutrino energy density at recombination through its effect on the expansion rate. The neutrino energy density at recombination is therefore the direct CMB observable, which does not depend only on  $N_\nu$ , but rather a combination of  $N_\nu$  and the neutrino temperature  $T_\nu$ . The equivalence between  $N_\nu$  and the value of  $N_{\text{eff}}^{\text{CMB}}$  derived from CMB observations assumes a standard neutrino thermal history. We refer to this “standard model” (SM) temperature of neutrinos as  $T_{\nu\text{SM}}$ . The equivalence between  $N_\nu$  and  $N_{\text{eff}}^{\text{CMB}}$  is broken if nonstandard processes take place after neutrino decoupling, resulting in a change in the neutrino temperature.

The possibility that the neutrinos might have a nonstandard temperature has a rich history and has been explored in numerous papers [4–11]. As an example of such a scenario, particles that decay after neutrinos decouple at a temperature of a few MeV may raise the photon temperature relative to the neutrinos, giving an effective neutrino temperature that is lower than that of the standard cosmological scenario. A similar effect can also occur for MeV-mass dark matter, which can stay in thermal equilibrium long enough to heat the photons relative to the neutrinos.

In this more general case, the statement that  $N_{\text{eff}}^{\text{CMB}} = N_\nu$  no longer holds true and is now generalized to

$$N_{\text{eff}}^{\text{CMB}} T_{\nu\text{SM}}^4 = N_\nu T_\nu^4. \quad (2)$$

On the right-hand side, we have the true neutrino number,  $N_\nu$ , and the true neutrino temperature,  $T_\nu$ . The total neutrino energy density (as long as the neutrinos are relativistic) is then proportional to  $N_\nu T_\nu^4$ . If an observer *assumes* a standard-model neutrino temperature,  $T_{\nu\text{SM}}$ , then  $N_{\text{eff}}^{\text{CMB}}$  is the neutrino number deduced from CMB measurements, or any other measurement derived from the neutrino energy density alone.

The degeneracy between  $N_{\text{eff}}^{\text{CMB}}$  and  $N_\nu$  can be disentangled by combining observables from both the CMB and BBN. For example, Nollett and Steigman recently used the combination of BBN abundance predictions and CMB

measurements to constrain electromagnetically coupled dark matter particles that raise the photon temperature relative to that of the neutrinos [10]. They also showed in [11] that the opposite effect can be accomplished if a coupling is introduced between the dark matter sector(s) and neutrinos.

In this paper, we consider the most general possible case, in which  $N_\nu$  and  $T_\nu$  are treated as free parameters, and then determine the observational constraints from a combination of BBN and the CMB. In section II we discuss how  $T_\nu$  and  $N_\nu$  affect BBN and CMB observables while in section III we explore these effects more precisely using numerical simulations of BBN and its effects on primordial abundance predictions of  $^4\text{He}$  and deuterium. Combining these results with observational limits on the primordial element abundances, we derive corresponding limits on  $T_\nu$  and  $N_\nu$  and then examine the effects on  $N_{\text{eff}}^{\text{CMB}}$ . We discuss our conclusions in section IV. In summary, we find that even tight observational bounds on  $^4\text{He}$  and deuterium can be consistent with one extra sterile neutrino for a change in  $T_\nu$  from the standard model value of only  $\sim -5\%$ , while two additional sterile neutrinos are ruled out.

## II. THE EFFECTS OF $N_\nu$ , $N_{\text{eff}}^{\text{CMB}}$ AND $T_\nu$ ON THE CMB AND BBN

Currently, the number of neutrinos  $N_\nu$  can be probed in two separate eras of cosmic evolution, both producing distinct and independent observables. These two eras are (i) *Big Bang nucleosynthesis* (BBN), when light nuclear elements are produced and (ii) *Recombination*, when electrically neutral atoms first form allowing photons to free-stream and produce the cosmic microwave background (CMB) radiation. These two eras differ by orders of magnitude in energy, with BBN occurring around  $\sim 1$  MeV and CMB decoupling occurring at the eV scale.

Just before BBN, another important process, *neutrino decoupling*, takes place when the neutrinos drop out of thermal equilibrium at a temperature of around  $2 - 3$  MeV. In the standard scenario, when the neutrino sector decouples from the thermal background, it inherits the temperature  $T_\nu$  from the thermal bath. Then  $T_\nu$  is solely affected by the expansion of the universe, scaling as  $T_\nu \propto a^{-1}$ , where  $a$  is the cosmic scale factor.

When the temperature drops below the mass of the electron,  $e^+e^-$  pairs annihilate, heating the photons relative to the neutrinos, and producing a final ratio of

$$T_{\nu\text{SM}} = (4/11)^{1/3} T_\gamma. \quad (3)$$

The assumption made in this scenario is that there are no processes that modify either the neutrino temperature or the photon temperature between neutrino decoupling and BBN. We refer to this evolution as the standard model evolution, hence our use of the SM subscript.

Note that the neutrinos are partially heated by the  $e^+e^-$  annihilations. This effect is usually absorbed into the definition of  $N_\nu$  rather than  $T_\nu$ , giving an effective neutrino number of  $N_{\text{eff}} = 3.046$  [12, 13]. However, this effect is very small compared to the large changes in  $N_\nu$  and  $T_\nu$  considered here, so we ignore it in what follows.

The CMB measurements are sensitive to the total energy density at the epoch of recombination; the neutrino energy density enters into this calculation only through its total energy density, which is simply proportional to  $N_\nu T_\nu^4$ . If we allow both  $N_\nu$  and  $T_\nu$  to have values that differ from their standard-model values, then the number of neutrinos inferred from CMB measurements will be

$$N_{\text{eff}}^{\text{CMB}} = N_\nu (T_\nu / T_{\nu\text{SM}})^4. \quad (4)$$

We can therefore quantify a possible shift in the neutrino temperature through the ratio of  $T_\nu / T_{\nu\text{SM}}$ , where we also use the superscript CMB to indicate the era that the parameter  $N_{\text{eff}}$  is probing.

The processes involved in Big Bang Nucleosynthesis also depend on both  $N_\nu$  and  $T_\nu$ , but in a more complex way than the CMB observables do. (For a recent review of BBN, see, e.g., Ref. [14]). First, the element abundances depend on the expansion rate during BBN given by

$$H^2 = \frac{8\pi G}{3} (\rho_\gamma + \rho_{e^+e^-} + \rho_{\nu\bar{\nu}}), \quad (5)$$

where the three terms that contribute to the total energy density are those of the photons, electron-positron pairs and neutrinos, respectively. The neutrino term includes the contributions of all three standard-model neutrinos as well as any possible nonstandard (e.g., sterile) neutrinos. If the expansion rate was the only place where the neutrino temperature entered into the BBN calculations, then the primordial element abundances would only depend on the total neutrino energy density, just like the CMB observations. That this is not the case is the fundamental reason we can break the degeneracy between  $N_{\text{eff}}^{\text{CMB}}$  and  $N_\nu$ .

Beyond its role in the expansion rate during BBN, the neutrino temperature also plays a crucial role in the weak interaction rates, which determine the light element abundances. Down to temperatures of  $\sim 1$  MeV, the protons and

neutrons are kept in thermal equilibrium via the following weak interaction processes

$$\begin{aligned} n + \nu_e &\leftrightarrow p + e^-, \\ n + e^+ &\leftrightarrow p + \bar{\nu}_e, \\ n &\leftrightarrow p + e^- + \bar{\nu}_e. \end{aligned} \tag{6}$$

The total rates for the conversion of neutrons to protons and protons to neutrons are

$$\begin{aligned} \lambda_{n \rightarrow p} &= A \int_{m_e}^{\infty} dE_e \frac{E_e |p_e|}{1 + \exp[E_e/kT_e]} \\ &\times \left\{ \frac{(E_e + Q)^2}{1 + \exp[-(E_e + Q)/kT_{\nu_e}]} + \frac{(E_e - Q)^2 \exp(E_e/kT_e)}{1 + \exp[(E_e - Q)/kT_{\nu_e}]} \right\}, \end{aligned} \tag{7}$$

and

$$\lambda_{p \rightarrow n} = \lambda_{n \rightarrow p}(-Q), \tag{8}$$

respectively, where  $Q = m_n - m_p$ , and the subscripts  $e$  and  $\nu_e$  denote the quantities associated with the electron and the electron-neutrino, respectively. The constant  $A$  is determined from the requirement that  $\lambda_{n \rightarrow p}(T, T_{\nu_e} \rightarrow 0) = 1/\tau_n$  (the neutron decay rate).

The important point is that these weak rates are sensitive to the neutrino temperature  $T_\nu$ , where we assume that neutrino mixing gives all three neutrinos the same temperature. (The actual effect on the various element abundances is discussed in detail in the next section). This dependance on  $T_\nu$  allows a combination of BBN abundance predictions and CMB observations to yield complementary limits on  $T_\nu$  and  $N_\nu$  when these quantities are varied independently.

Note that the measurable quantity that affects both BBN and the CMB is not the absolute value of  $T_\nu$ , but the ratio of  $T_\nu$  to  $T_\gamma$ , since all calculations for BBN and the CMB are scaled off of the background photon temperature. For simplicity, we treat any change in  $T_\nu/T_\gamma$  as an effective change in  $T_\nu$ . However, note that  $T_\nu/T_\gamma$  can be altered by changing *either*  $T_\nu$  or  $T_\gamma$ .

Models in which the neutrino temperature (or more precisely,  $T_\nu/T_\gamma$ ) is modified from its standard value can be divided into two broad categories. If the neutrino temperature changes after decoupling but before BBN, then  $T_\nu$  at BBN is the same as for the CMB analyses, and both differ from the standard model value. This is the scenario we envision here. There are many such models that can alter the value  $T_\nu$  from the standard model value during BBN; see Refs. [4–11] for examples. A second possibility, not examined here, is that  $T_\nu$  takes its standard model value during BBN, but then changes between BBN and decoupling. For example, entropy release between BBN and decoupling results in an increase in  $T_\gamma$ , so an effective decrease in  $T_\nu$  at a given value of  $T_\gamma$ . These scenarios are straightforward to analyze: BBN proceeds in the standard way, but the CMB limits are altered by changing the value of  $N_{\text{eff}}^{\text{CMB}}$  relative to  $N_\nu$  as given by Eq. (4). We will not examine such scenarios in detail here.

The two observables we discuss here (BBN and CMB) are not the only means by which the degeneracy between  $N_\nu$  and  $T_\nu$  can be broken. A third possibility involves large-scale structure constraints on the neutrino mass. Since these observations are based on an epoch at which the neutrinos have become nonrelativistic, they are actually sensitive to the quantity  $T_\nu^3 \Sigma_\nu m_\nu$ , where the sum is over all three types of neutrinos, again assumed to have a single common temperature  $T_\nu$ . A discussion of these limits and their relation to CMB and BBN limits is beyond the scope of this paper, but see the analysis in Ref. [7].

### III. NUMERICAL RESULTS, OBSERVATIONAL BOUNDS, AND COMBINED BBN/CMB CONSTRAINTS

In order to investigate the interplay between a varying neutrino number  $N_\nu$  and neutrino temperature  $T_\nu$  on light element abundances from BBN, we solve the rate equations and cosmological evolution numerically and present our results in this section. We used the computer code *AlterBBN* [15] originally written by A. Arbey, and later modified by K. P. Hickerson [16]. Our version, modified from Hickerson's version 1.6, along with a supplementary explanation of our analysis is available at [17].

In our simulations, we allow  $N_\nu$  and  $T_\nu$  to vary separately. We take  $T_\nu$  to be the same for all three standard-model neutrinos and any additional sterile neutrinos, which will be the case as long as there is sufficient mixing between all of the neutrino sectors. This corresponds, for instance, to the most interesting cases of mixing with sterile neutrinos to provide a possible explanation for the tension in short-baseline neutrino oscillation experiments (see, e.g., Ref. [7] and references therein). We also assume that the neutrino temperature obeys the standard evolution after BBN, i.e.,

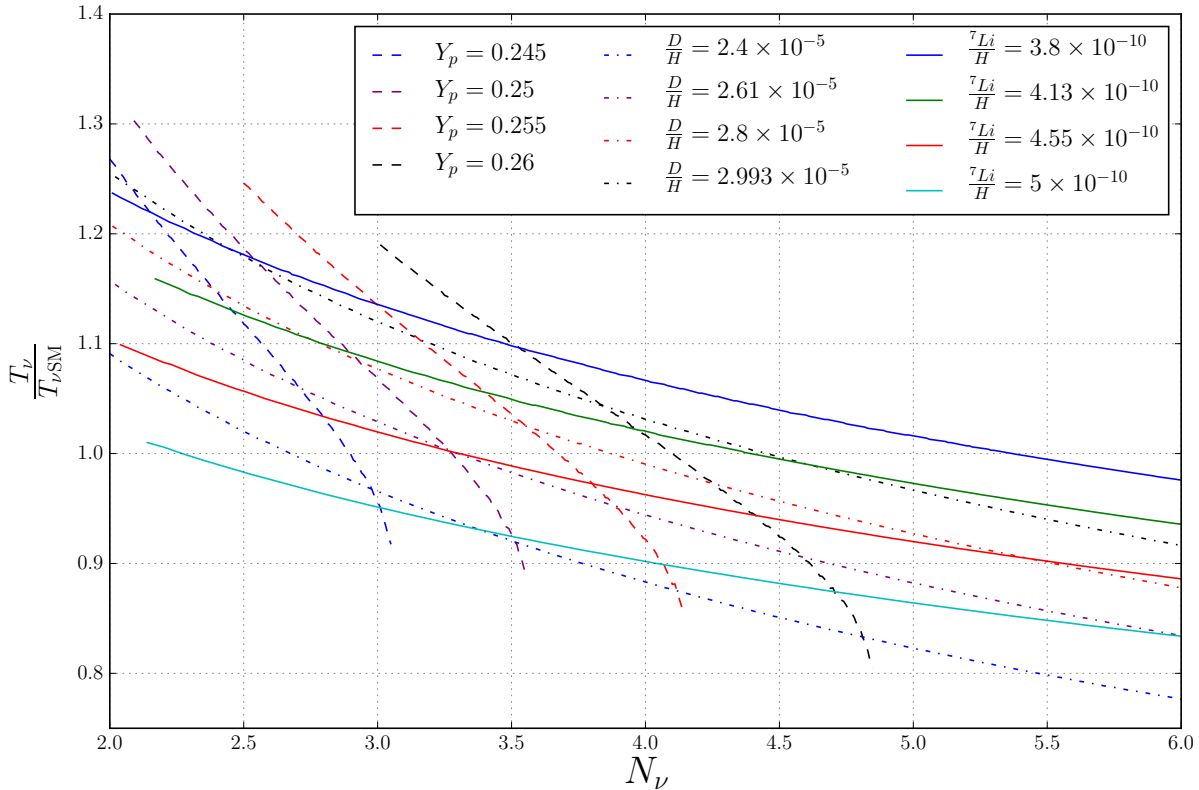


FIG. 1: Predicted primordial abundances of  ${}^4\text{He}$ , deuterium, and  ${}^7\text{Li}$  in the plane defined by the neutrino number,  $N_\nu$ , and the ratio of the neutrino temperature to its standard-model value,  $T_\nu/T_{\nu\text{SM}}$ .

it decreases as the inverse of the scale factor. One may consider nonstandard deviations from both of these scenarios for  $T_\nu$ ; however, we leave such possibilities for future work and focus on the scenario listed above.

In our simulations we take a baryon to photon ratio of  $\eta = 6.19 \times 10^{-10}$  and a neutron lifetime of  $\tau_n = 880.3$  seconds [18], and derive the primordial element abundances as a function of two parameters:  $N_\nu$ , and a nonstandard neutrino temperature  $T_\nu$ . We parametrize the shift of the neutrino temperature relative to the standard-model neutrino temperature as the ratio  $T_\nu/T_{\nu\text{SM}}$ . In order to keep the results as general as possible, we do not assume a particular model or mechanism for the nonstandard value of  $T_\nu$ ; instead, we assume that the neutrino temperature differs by a constant factor from the standard model value.

Since we assume three standard neutrinos plus an undefined additional contribution to  $N_\nu$ , it is reasonable to take  $N_\nu \geq 3$  to be a physical lower bound. Note, however, that there are brane-world scenarios which achieve a negative change in the relativistic energy density [19], so for completeness we allow  $N_\nu$  to vary in the range  $2 \leq N_\nu \leq 6$ .

In Fig. 1, we show the primordial  ${}^4\text{He}$  mass fraction,  $Y_p$ , and the deuterium and  ${}^7\text{Li}$  number densities relative to hydrogen, as a function of  $N_\nu$  and  $T_\nu/T_{\nu\text{SM}}$ . For the standard model value of the neutrino temperature,  $T_\nu/T_{\nu\text{SM}} = 1$ , we obtain the familiar result that  $Y_p$  increases with  $N_\nu$ , because the increased expansion rate causes the weak rates to freeze out at a higher temperature, resulting in more neutrons, and nearly all of these neutrons (modulo free neutron decay) end up bound into  ${}^4\text{He}$ . The deuterium abundance also increases with  $N_\nu$  at fixed  $T_\nu$ , as the increased expansion rate allows less time for the deuterium to fuse into heavier elements. On the other hand, the  ${}^7\text{Li}$  abundance decreases with increasing  $N_\nu$ .

The effect of altering  $T_\nu$  at fixed  $N_\nu$  is not as obvious, because an increase in  $T_\nu$  results in both an increase in the weak rates and an increase in the expansion rate. From Fig. 1, we see that increasing  $T_\nu$  at fixed  $N_\nu$  results in a net increase in  $Y_p$ , indicating that the effect of increasing the expansion rate (which increases  $Y_p$ ) dominates the effect of increasing the weak rates (which decreases  $Y_p$ ). Note, however that these two effects begin to cancel for  $T_\nu/T_{\nu\text{SM}} < 0.9$ , at which point  $Y_p$  begins to become nearly insensitive to  $T_\nu/T_{\nu\text{SM}}$ . The behavior of deuterium and

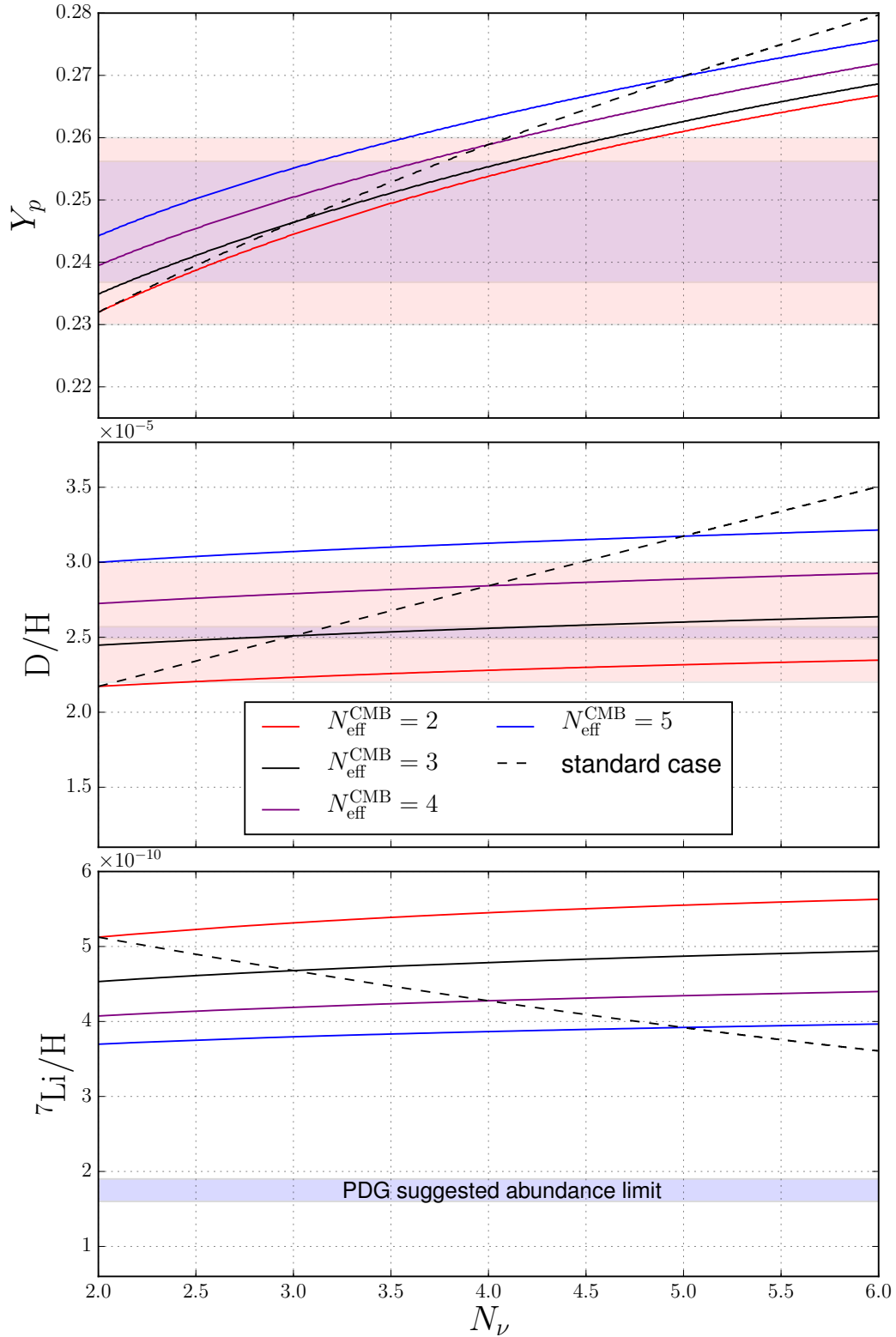


FIG. 2: Predicted primordial abundances of  $^4\text{He}$ , deuterium, and  $^7\text{Li}$  as a function of the number of relativistic neutrinos  $N_\nu$  when the neutrino temperature is allowed to vary from its standard-model value. The neutrino temperature is parametrized in terms of  $N_{\text{eff}}^{\text{CMB}}$  as defined in Eq. (4); solid curves give abundances for the indicated values of  $N_{\text{eff}}^{\text{CMB}}$ . Dashed curve gives abundances for the standard-model neutrino temperature. Light shaded region corresponds to the conservative abundance limits from Eqs. (9)-(10); dark shaded region gives the stricter PDG limits from Eqs. (11) - (12).

${}^7\text{Li}$  is much more straightforward, since these two nuclides are primarily sensitive to the overall expansion rate. Thus,  $D/H$  increases with the increased expansion rate produced by an increased value of  $T_\nu/T_{\nu SM}$ , while  ${}^7\text{Li}$  decreases.

Note that while  $T_\nu/T_{\nu SM}$  and  $N_\nu$  are the two parameters that enter directly into the BBN calculation, they are not the most useful to use in our analysis. Instead, we take  $N_{\text{eff}}^{\text{CMB}}$  to be one of our parameters, as this is the effective number of neutrinos measured by the CMB. Eq. (4) then leaves only one free parameter, which we can take to be either the neutrino number or temperature. Since it is the neutrino number which is the physically relevant quantity, we adopt it as our second parameter. The value of  $T_\nu/T_{\nu SM}$  can then be determined from Eq. (4).

Throughout our analysis we adopt two sets of observational limits for the primordial  ${}^4\text{He}$  mass fraction,  $Y_p$ , and the deuterium to hydrogen ratio  $D/H$ . We define a “conservative” set of bounds:

$$D/H = (2.6 \pm 0.4) \times 10^{-5}, \quad (9)$$

$$Y_p = 0.245 \pm 0.015, \quad (10)$$

while in addition using the tighter bounds suggested by the Particle Data Group (the “PDG” bounds) [18]:

$$D/H = (2.53 \pm 0.04) \times 10^{-5}, \quad (11)$$

$$Y_p = 0.2465 \pm 0.0097. \quad (12)$$

Note that the CMB limit of equation (1) results in a band in the  $N_{\text{eff}}^{\text{CMB}}$ ,  $N_\nu$  parameter space. We consider this limit to be self-evident and do not depict it in our figures. Note also that the exact bound depends on the datasets combined with PLANCK’s observations. We therefore allow the reader to select a preferred range for  $N_{\text{eff}}^{\text{CMB}}$ .

In Fig. 2, we present the predicted element abundances as a function of  $N_\nu$ , for a variety of  $N_{\text{eff}}^{\text{CMB}}$  values (solid curves). The dashed curve corresponds to the standard temperature case,  $T_\nu/T_{\nu SM} = 1$ . As is clear in the figure, this curve intersects each  $N_{\text{eff}}^{\text{CMB}}$  curve at the point  $N_{\text{eff}}^{\text{CMB}} = N_\nu$ . Curves of constant  $N_{\text{eff}}^{\text{CMB}}$ , as defined in Eq. (4), correspond to curves of constant neutrino energy density. Thus, tracing the element abundances along each solid curve allows us to see the effect of changing both the neutrino temperature and number in such a way that the neutrino energy density is unchanged. In this case, the only effect on the primordial element abundances comes from the change in the weak rates. Decreasing  $N_\nu$  at fixed  $N_{\text{eff}}^{\text{CMB}}$  corresponds to increasing  $T_\nu$ . In the case of  ${}^4\text{He}$ , for example, this results in a decrease in  $Y_p$ , since the increased neutrino temperature increases the weak rates, allowing them to stay in thermal equilibrium longer and reducing the final neutron abundance.

Now consider the observational bounds. Allowing both  $N_\nu$  and  $N_{\text{eff}}^{\text{CMB}}$  to vary independently, we see that the conservative  $D/H$  bound is the least restrictive:  $N_\nu$  can have any value in the range we have investigated ( $2 \leq N_\nu \leq 6$ ), while  $N_{\text{eff}}^{\text{CMB}}$  can vary between 2 and 4. However, the PDG bounds on  $D/H$  place the tightest constraints on  $N_{\text{eff}}^{\text{CMB}}$ , namely  $N_{\text{eff}}^{\text{CMB}} \approx 3$  and  $3 \leq N_\nu \leq 4$ . Primordial helium provides the usual upper bound on  $N_\nu$ . Varying the neutrino temperature relaxes this bound somewhat, but even for  $N_{\text{eff}}^{\text{CMB}}$  as low as 2, we still have the upper bound  $N_\nu < 5$ .

In Fig. 3 we combine the deuterium and  ${}^4\text{He}$  limits to derive overall BBN constraints in the  $N_\nu$ ,  $N_{\text{eff}}^{\text{CMB}}$  plane. This figure shows the complementarity of these two sets of limits, with deuterium giving the upper bound on  $N_{\text{eff}}^{\text{CMB}}$  and  $Y_p$  giving the upper bound on  $N_\nu$  over most of the allowed region. An interesting feature of these limits is that even when we impose the CMB constraint of  $N_{\text{eff}}^{\text{CMB}} \approx 3$  with the restrictive abundance limits of Ref. [18], the value of  $N_\nu$  can be as large as 4, thus allowing on additional sterile neutrino. On the other hand, even with our less restrictive abundances limits and an arbitrary value for  $N_{\text{eff}}^{\text{CMB}}$ , a value of  $N_\nu = 5$  (two additional sterile neutrinos) is ruled out. Fig. 3 illustrates the additional constraining power of BBN beyond what is available with the CMB alone. When  $T_\nu$  is allowed to vary freely, a given value of  $N_{\text{eff}}^{\text{CMB}}$  from the CMB provides no constraint on  $N_\nu$ . But adding the BBN constraint gives an upper bound on  $N_\nu$  for any value of  $N_{\text{eff}}^{\text{CMB}}$ . As noted previously, this upper bound is derived primarily from limits on  ${}^4\text{He}$ , rather than deuterium.

The allowed regions from Fig. 3 are displayed in more detail in Fig. 4, where we now superimpose a heat map to illustrate the corresponding value of  $T_\nu/T_{\nu SM}$ . The region corresponding to  $N_{\text{eff}}^{\text{CMB}} \approx 3$ ,  $N_\nu > 3$  corresponds to a value of  $T_\nu$  smaller than its standard-model value. Interesting effects are achieved with a very small change in  $T_\nu$ . For instance, the point corresponding to  $N_{\text{eff}}^{\text{CMB}} = 3$  and  $N_\nu = 4$  corresponds to a  $\sim 5\%$  decrease in  $T_\nu$  relative to its standard model value.

It is well-known that the current BBN predictions for the primordial  ${}^7\text{Li}$  abundance differ significantly from the observationally-inferred values, with the BBN predictions a factor of 3 or more above the observed values. This has been dubbed the “lithium problem” (see, e.g., Ref. [20] for a recent review). It is therefore interesting to see whether the joint variation of  $N_\nu$  and  $T_\nu$  can ameliorate or solve this problem. It is clear from Fig. 1 that although some combinations of  $N_\nu$  and  $N_{\text{eff}}^{\text{CMB}}$  can reduce the predicted  ${}^7\text{Li}$  abundance, this reduction is short of what is needed to close the gap between prediction and observation. Furthermore the largest reductions in the predicted abundance lie in regions of parameter space that are excluded by the deuterium and  ${}^4\text{He}$  observations.

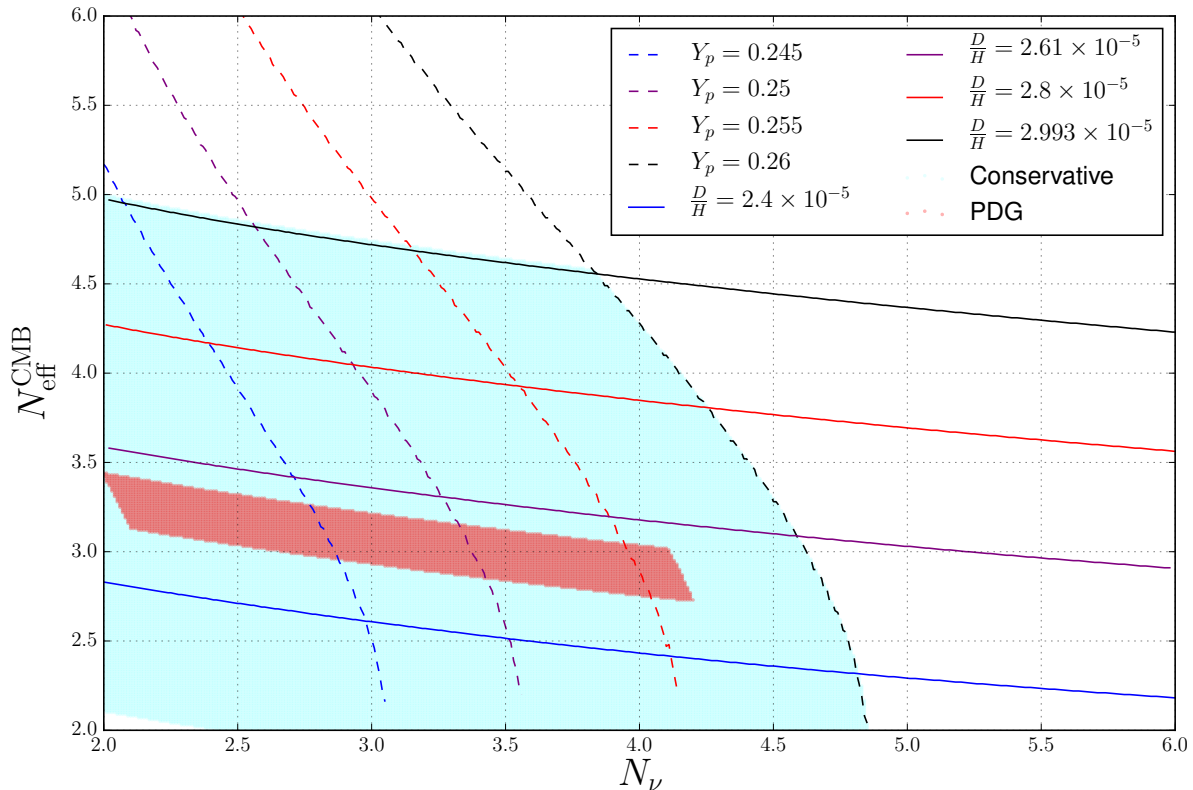


FIG. 3: Allowed region in the  $N_\nu$ ,  $N_{\text{eff}}^{\text{CMB}}$  plane given by combined observational limits on deuterium and  $^4\text{He}$ . Blue (light) region is allowed by the conservative bounds of Eqs. (9) and (10); red (dark) region is allowed by the more restrictive bounds in Eqs. (11) and (12). Dashed and solid curves correspond to the indicated  $^4\text{He}$  and deuterium abundances, respectively.

#### IV. CONCLUSIONS

While observations of the cosmic microwave background yield the most precise limits on cosmological parameters, our results show that Big Bang nucleosynthesis remains an indispensable tool. For models in which the neutrino number and temperature can both vary, the CMB alone cannot produce any limits on  $N_\nu$ , while a combination of the CMB and BBN yields a very useful bound.

In the models examined here, a value of the neutrino number as determined from the CMB of  $N_{\text{eff}}^{\text{CMB}} \approx 3$  can be consistent with a true neutrino number,  $N_\nu$ , as large as 4, thus allowing for an additional sterile neutrino. Such a model requires a reduction in the neutrino temperature of approximately 5% relative to the standard model neutrino temperature. However, a value of  $N_\nu = 5$  is ruled out for any value of  $N_{\text{eff}}^{\text{CMB}}$ .

The obvious direction for future investigation would involve more complex behavior for the evolution of the neutrino temperature, both during and following BBN. Some of these types of behavior have been discussed previously in Refs. [4–11], but these studies do not by any means exhaust all of the interesting possibilities.

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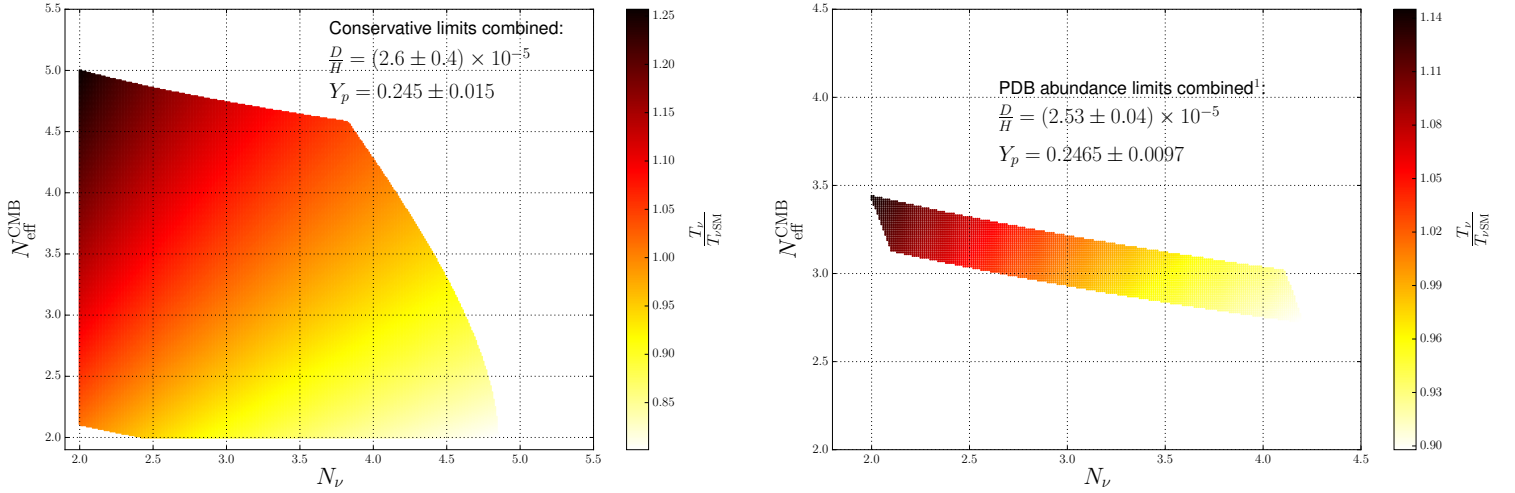


FIG. 4: The allowed regions in the  $N_\nu$ ,  $N_{\text{eff}}^{\text{CMB}}$  plane from Fig. 3, showing the corresponding value of  $T_\nu/T_{\text{SM}}$  as a heat map.

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